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Exploring the Intersection of Fuzzy Logic and Quantum Logic: A New Frontier in Non-Classical Logics

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Abstract

Fuzzy logic can handle ambiguity and uncertainty. Quantum physics led to the development of quantum logic, which, unlike fuzzy logic, is based on projectors' vector subspaces. The two theories' relationship is fascinating. These non-classical logics provide viable solutions for handling intricacy, uncertainty, and information processing in a variety of disciplines. The applications of fuzzy-quantum logics in artificial intelligence, decision-making, control systems, and quantum computing are covered in this article along with possible future research paths in the field. We also look at the evolution of quantum fuzzy information theory as well as the application of quantum logic theory to fuzzy logic and other non-classical logics. We also stress the potential of mixed algorithms and optimisation methods based on fuzzy-quantum logics to address challenging issues. We wrap up by highlighting the importance of additional study in these fields and summarising the overall results. The suggested study directions could help non-classical logics progress by producing fresh ideas, theories, and applications that might have an effect on a number of different disciplines and stimulate innovation there.

Keywords: Fuzzy Logics, Quantum Logics, Fuzzy-Quantum Logics, Quantum Logic Theory.

I. INTRODUCTION

Computer scientists often formalise fuzzy logic. Zadeh's work inspired it. It helps us deal with uncertainty. Fuzzy logic uses t-norms and t-conorms for set intersection and union. Quantum logic uses quantum physics' framework. This approach is based on projector-identified vector subspaces, not membership values. All projector lattices allow conjunction and disjunction. The two theories' relationship is fascinating. A fuzzy membership result is formed when a projection interacts with such a normalised vector. In certain circumstances, the confluence among projectors corresponds to the t-fuzzy norm's algebraic product. Quantum logic considers projectors, unlike fuzzy logic's fuzzy sets. We can solve the algebraic product's idempotence problem. If projectors are mutually commuting, we get Boolean logic. Quantum logic help us grasp algebraic product and fuzzy norm sum semantics.

In computer programming, a connective of quantum logic (abbreviated QL) is crucial. The ideas of multivalued fuzzy function have been investigated in the construction of multi-valued logic circuits. When dealing with hazy concepts, fuzzy sets and fuzzy logic provide the fundamental foundation. The traditional binary negation, conjunction, disjunction, and implication are all expanded in fuzzy logic to mappings that accept values in the unit interval. Typically, a fuzzy negation is used to simulate a fuzzy negation operator. A triangular norm or a conjunction just on unit interval are often used to represent a fuzzy conjunction operator (in short, t-norm). In most cases, a triangular co norm is used to simulate a fuzzy disjunction operator (t-conorm for short). A fuzzy implication operator may be modelled using a variety of techniques. It may be built using one of many parameterized generating functions or one of the other three fuzzy logic operators.

II. CONCEPT OF FUZZY LOGICS

In our daily lives, we may encounter circumstances in which we are not able to tell whether a given condition is true or untrue. Something that is fuzzy is ambiguous or hazy. AI's fuzzy logic offers useful flexibility in thinking.

Fuzzy logic: Fuzzy logic mimics human thought (FL). This is how people decide. It includes "YES" and "NO" third-party choices.

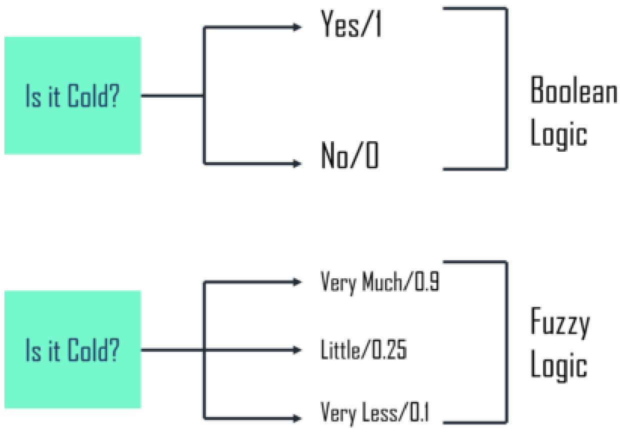

Figure 1: ***Fuzzy logic mimics human thought***

A computer may comprehend a logic block that accepts precise input and produces TRUE or FALSE, like YES or NO. In contrast with computers, individuals have more possibilities between YES and NO, such like:

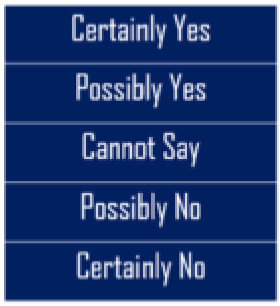

Figure 2: ***Individuals with possibilities between YES and NO***

Fuzzy logic utilises the levels of possible inputs to give a clear answer. Now, think about how this reasoning works in the real world:

It can be used in a wide range of systems, from workstation-based systems to huge networks to microcontrollers.

It can also be employed in hardware, software, or a combination of the two.

Fuzzy logic is used because... In general, we use the system of fuzzy logic both for practical and business purposes. For example, it controls machines as well as consumer goods; if it doesn't give exact reasoning, it gives good reasoning; and it helps engineers deal with ambiguity. Now that you know how fuzzy logic is used in AI and why, let's talk about how it works.

2.1. A Quantum Implementation of Conjunction and Disjunction in Fuzzy Logic

This section shows how applying fuzzy standardized procedures (Union, Intersection, and Maximum, as well as Alpha-cut) to set theory can be used to solve QUBO problems placed above a white underlying qfuzzy systems, and so can be done on quantum computers.

Humans successfully characterise things using language phrases like tiny, old, long, and swift. But it's unlikely that sets of things that meet these linguistic requirements can be defined using traditional set theory. Consider for a moment that someone has been allocated to the group of tall people. A rule like "a person that is less that 1mm smaller than just a tall person is likewise tall" appears appropriate to create in order to define our set since if a second person is just marginally smaller, it must also be allocated to our set. If we constantly use this criterion, however, plainly people of any stature will be included in the group of tall people. Any standard for the notion tall will be difficult to justify. Conversely, it is simple to locate individuals who are tall and little, respectively. It is not a difficulty to model the common scenarios, however it is difficult to express the penumbra between both the ideas using classical sets [1].

This special compendium shows how fuzzy systems have a significant impact on quantum physics. It also provides comprehensive explanations of the evolution of fuzzy logic, from fuzzy sets through fuzzy systems, with the probability presentation for fuzzy systems emerging as the most significant outcome.

The research of quantum mechanics, that is a brand-new concept, makes use of the significant results on fuzzy systems. There are eight significant findings drawn. The author has shown the need of waves in mass-point movements in classical mechanics, demonstrating that every classical mass-point motion has waves mass-point dualism and that each microscopic particle motion has wave-particle dualism. This finding demonstrates the unification of classical mechanics with quantum mechanics.

2.2. Quantum mechanics to quantum logic

After W. Heisenberg, M. Born, and P. Jordan found "matrix mechanics" in 1925 and E. Schrodinger came up with "wave mechanics" in 1927, it took John von Neumann two years to figure out the math behind quantum mechanics. (1927–1929). He knew that the context is Hilbert space operators. Each quantum mechanical system has a separate Hilbert space that is described by unitary vectors called xH. Observable means measurable. Most of the time, self-adjoint operators on Hilbert spaceH do not commute and are not limited. John von Neumann then looked at linear operators on Hilbert spaces. In 1929, the spectral hypothesis on unbounded self-adjoint operators was the high point of this work [2].

In the late 1980s, fuzzy set models of quantum logics were proposed because there were formal similarities between fuzzy set operations and order-theoretic quantum logic operations [pykacz, 1987]. Fuzzy set complementation, which gets rid of Zadeh's unions and crossings [Pkacz, 1992], should be used to model Orth complementation. This was clear right away. Birkhoff-von Neumann quantum logic is based on fuzzy set equivalents of excluded middle and contradiction in point (c) [3].

The author has been attempting to create fuzzy set version of quantum logics since 1987, but his prior attempts required actual fuzzy set operations, which he was unable to implement. Fuzzy set theory does not apply to the union (sum) two fuzzy sets, and Lukasiewicz intersection (product) two fuzzy sets, or (a), which are all point - wise algebraic additions over membership functions. A null set being the every fuzzy set that really is discontinuous amongst it-self was introduced as a natural constraint in 1994 [Pkacz, 1994], which amended the third among Mgczyliski's [1973a; 1974] basic requirements. It's important to look at how these processes fit together and how they make sense.

2.3. Fuzzy Quantum Structures

A current trend in logico-algebraic quantum mechanics is to study orthoalgebras as well as effect algebras instead of orthomodular posets and lattices. It makes sense to think about representing these structures with fuzzy sets as well as propositional functions that use the right operations. In 1996 and 1997, DvureEenskij made progress in this area by adding up membership functions in an algebraic way [4, 5]. In this chapter, these results are extended to fuzzy sets, which also include operations called "conorms" that are explained by two pairs of tri-angular standards, and also Lukasiewicz's work in these models is looked at.

2.4. Quantum probability fuzzy-set models

Some quantum system tests do not fit within the numbers permitted by classical (Kolmogorov) probability theory. Such occurrences, which are frequently associated with Bell's inequalities, imply that quantum mechanical probability computations must be altered. Even a cursory examination of "quantum probability" would be too excessive for this essay. We will concentrate on quantum-logical approaches to this problem. The Kolmogorov triple (R,T,P) is employed in the logics of quantum theory approaches to the theoretical underpinnings of quantum mechanics.

A pair (L,P) of an ortho-modular posets (or quantum logic) and a probability measurement (state) replaces F of a random subset R and a probability measure P. Probability endeavors in quantum logic are nonnegative, normalized, and limited to families of pairs disjoint (or "traditional" quantum logic) components. The term makes this quite evident. The probability calculus of Kolmogorov, which is based on Boolean algebra, is used to investigate probability rather than theory of quantum mechanics. This is especially proof when employing quantum logics, because all relevant theorems explain how quantum logic probability measures may satisfy Bell-type inequalities as long even as quantum logic is just a Boolean algebra [6, 7].

Quantum random events are not subsets of fundamental events. Instead, they are mathematical objects in and of themselves. In an abstract model, they are orthomodular poset components, and in a researcher Hilbert space logical model, they are always closed subspaces (or orthogonal projections). This makes it difficult to consider quantum random occurrences as subsets within phase space.

quantum logic shows all pure states of a system of concepts by probability measurements on its logics, forming a convex set. In the phase space description of a classical statistical system, Borel subsets relate the logic to a Boolean algebra, and pure states are Dirac measures centred with one subset, linking them to system points. Borel subsets of the phase space, which are logic components, may be related to random events since physical system attributes produce random events. In classical set theory, disjunctions and conjunctions of physical system statements are utilised to produce unions and crossings of random occurrences [8, 9].

By substituting fuzzy subsets of the unique properties of quantum systems for the conceptual quantum logics found inside the fundamental principles of the quantum physics probability calculus, it is possible to create a flawless parallel among both Kolmogorov's probability calculus, which is applicable to conventional statistical processes, as well as the quantum probability calculus, which is applicable to quantum systems. Intersections & unions are composed of system pieces that either are connected or not. Despite the fact that this logic is a lattice, its conclusions do not necessarily match quantum logic using fuzzy sets, unlike classical statistical physics. As a result, it may be difficult to create "compound" quantum random occurrences using joins and meets rather than Lukasiewicz's unions and intersections [12].

To construct a fuzzy probability theory, fuzzy unions & intersections can be utilised instead of Lukasiewicz's approaches. Several investigations have produced a fuzzy probability theory. The majority of these articles employ the traditional Zadeh operations, which, as Gonseth [1938] pointed out within the context of the many logic, do not meet the false dilemma rule and also the rule of contradiction proof for just any actual fuzzy set when paired with ordinary fuzzy set complementation [13, 14].

2.5. Case study

Quantum mechanics and fuzzy-set theory are combined mathematically to create quantum probability fuzzy-set models, which more flexibly reflect doubt and ambiguity than traditional probability theory. These models have been applied in a number of industries, including image analysis, banking, and decision-making.

Let's look at a case study in finance where stock price predictions were made using quantum probability fuzzy-set models. The dataset used in this research was made up of a specific stock's daily ending values over the previous two years. The objective was to create an algorithm that could forecast the stock price for the following day using the prices from the prior day.

The dataset was first converted into a representation of a fuzzy set, where each ending price was given a degree of membership to various fuzzy sets based on how closely the price matched up with particular preset values. Such a price would have a high degree of membership to the "average" fuzzy set and a lesser degree of membership to the "low" and "high" fuzzy sets. Another example would be a price that is close to the average of the previous values.

The next step was to create a quantum probability model using the fuzzy-set representation, where each fuzzy set was represented by a quantum state and the degree of inclusion was represented by the probability amplitude of the state. The characteristics of the quantum states and the procedures for changing the states in response to new data were then taught to the model using the historical data.

Each ending price can be written as a vector of membership degrees to various fuzzy sets in the dataset's fuzzy-set representation, as shown by the notation:

$$x = (x_1, x_2, \ldots, x_n)$$

where xi represents the degree of affiliation with the ith fuzzy collection. A fuzzy reasoning method based on the distance of the price to the centres of the fuzzy sets can be used to calculate these membership degrees.

The quantum probability model can then be built using a quantum state representation, in which each fuzzy set is connected to a quantum state that is depicted by a complex vector, written as follows:

$$|x_i> = a_i|0> + b_i|1>$$

where $|0>$ and $|1>$ are the basic states of a two-level quantum system, and a_i and b_i are complex probability amplitudes. (qubit).

The square modulus of the probability amplitude, represented by the notation: can be used to calculate the chance of sensing a specific quantum state.

$$P(x_i) = |a_i|^2 + |b_i|^2$$

The Schrodinger equation, which describes how the state vector transforms when a Hamiltonian operator acts upon it, can be used to explain how the development of quantum states over time. The fuzzy inference principles and historical data can be used to build the Hamiltonian, which can then be updated using a learning method like gradient descent.

As a weighted average of the potential results correlating to each quantum state, the expected value of the stock price for the following day can be calculated as follows:

$$E(x) = \sum i \; x_i P(x_i)$$

where the total of all potential quantum states is used. Standard measures like mean squared error or mean absolute error can be used to assess the model's precision, and statistical tests like t-tests or ANOVA can be used to compare it to other models.

The dataset of a specific stock's daily ending values over the previous five days, as shown below:

Table 1: ***Daily Ending Values Of Stock Over Five Days***

Date	Closing Price
2023-03-01	100
2023-03-02	105
2023-03-03	110
2023-03-04	95
2023-03-05	115

Figure 3: ***Graph showing daily ending values of stock over five days.***

Based on some predefined guidelines, we can use fuzzy sets to depict the extent to which each closing price belongs to various groups. For instance, we can define "low," "medium," and "high" fuzzy sets for the ending values, with the following membership functions:

Low: $\mu L(x) = max\left(0, 1 - \frac{x - 80}{20}\right)$

Medium: $\mu M(x) = max\left(0, min\left(\frac{x - 80}{20}, \frac{120 - x}{20}\right)\right)$

High: $\mu H(x) = max\left(0, \frac{x - 100}{20}\right)$

where the membership degrees range from 0 to 1 and x represents the final price.

The following table illustrates how we can calculate the degree of membership of each ending price to each fuzzy set using these membership functions:

Table 2: ***Degree of membership of each ending price***

Date	Closing Price	μL	μM	μH
2023-03-01	100	0.5	0.5	0.0
2023-03-02	105	0.0	0.5	0.5
2023-03-03	110	0.0	0.5	0.5
2023-03-04	95	0.5	0.5	0.0
2023-03-05	115	0.0	1.0	0.0

We can see that the first ending price belongs to the "low" and "medium" fuzzy sets to a degree of 0.5 and to the "high" fuzzy set to a degree of 0. The degree of membership for the second and third closing values to the "medium" and "high" fuzzy sets is 0.5, and the degree of membership for the "low" fuzzy set is 0.

Following the computation of each ending price's degree of inclusion in each fuzzy set, we can use these results to build a quantum probability model as previously discussed in the case study. Please be aware, though, that building a quantum probability model would actually be much more difficult and require extra stages like defining the Hamiltonian operator and running learning methods to keep the model current.

By measuring the quantum state associated with the current fuzzy-set representation and calculating the anticipated value of the price based on the probability amplitudes of the states, the model was then used to forecast the stock price for the following day. The model's precision was assessed using common measures like mean squared error and contrasted with that of other models like neural networks and traditional probability models [10].

In general, this case study demonstrates how quantum probability fuzzy-set models can be applied to real-world issues and highlights the possible advantages of using these models to identify ambiguity and uncertainty in complicated systems.

III. DIRECTIONS OF FUTURE STUDY

The principles of fuzzy logic and quantum logic, along with their combination in fuzzy-quantum logics and quantum logic theory, could be the foundation for a number of potential study paths. Potential research topics include, among others:

Applications for fuzzy-quantum logics have been suggested as a hybrid strategy that incorporates the benefits of fuzzy logic and quantum logic. Fuzzy-quantum logics have a variety of uses, including artificial intelligence, decision-making, and management systems, which could be the subject of future study.

Quantum logic theory is an extension of the logical theory that supports quantum physics. Future studies might look into how fuzzy logic and other non-classical logics can be incorporated into the idea of quantum logic. This might provide fresh perspectives on the origins of quantum physics and the essence of reality.

Quantum information theory is a discipline that studies the characteristics of information in the quantum world. Future study could explore the inclusion of fuzzy logic and quantum logic in information theory and create a unified paradigm for processing information in both conventional and quantum systems.

Fuzzy-quantum algorithms and optimisation techniques have been suggested as a means of resolving complex issues that are challenging to address with traditional algorithms. The creation of hybrid algorithms and optimisation methods that are based on fuzzy-quantum logics and quantum logic theory may be the subject of future study.

Developing computers based on the principles of quantum physics is the goal of the emerging area of quantum computing. Future studies could look into the possible uses of quantum computing in fuzzy, quantum, and fuzzy-quantum logics, as well as the use of these non-classical logics to create quantum algorithms and quantum information processing methods.

In general, the fusion of fuzzy logic and quantum logic has the potential to open up new study directions and could result in substantial improvements in our comprehension of complex systems and phenomena [11].

IV. CONCLUSION

The relationships between abstract quantum logics, various abstract quantum structures, as well as the various fuzzy set categories that make it up Lukasiewicz's unions and intersection are examined in this article. These connections frequently resemble an abstract of the link among Boolean algebras and regular (or "crisp") sets. In the traditional Kolmogorovian possibility calculus, sharp sets of random encounters are used to construct the Boolean algebras that specify the probability actions. Probabilities are defined in quantum probabilistic calculus according to how general they are. These probabilities are derived from various quantum structures, which are composed of fuzzy sets containing quantum random occurrences.

The only fuzzy set construction techniques that can be utilised to create the quantum frameworks that physicists are interested in are Lukasiewicz's union and intersection. To determine which pair would function best as an algebraic characterization of logical conjunction and disjunction, it was required to consider how order-theoretic meet to join and how Lukasiewicz's union and intersection may be utilised to create fuzzy set models of quantum logics. This needs more research. The meet and join approach, which has been around for 60 years, is seen to be outdated and should be replaced with Lukasiewicz's procedures.

Finally, it can be said that the fusion of fuzzy logic and quantum logic, as well as their mixed methods in fuzzy-quantum logics and quantum logic theory, offers fascinating prospects for future study. In a variety of areas, such as artificial intelligence, making decisions, control systems, and quantum computing, these non-classical logics provide distinctive viewpoints and methods for addressing ambiguity, complexity, and information processing. Fuzzy-quantum logic, quantum fuzzy theory of information, hybrid algorithms, and optimisation strategies may all be investigated in order to gain new knowledge and improve our comprehension of complicated systems and phenomena. The advancement of new theories, approaches, and uses in these fields could have a significant effect on a number of academic subjects and spur creativity in the non-classical logics field.

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Conflicts of Interest

The authors declare no conflict of interest.

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